Lecture 2

Compound Propositions, Truth Table

Compound Propositions

Propositions can be denoted by **propositional variables** such as $p, q, r, s,$ etc.

Examples:

$p =$ New Delhi is the capital of India.

$q =$ 45 is a prime number.

We will now focus on producing new propositions from existing propositions.

Definition: A **compound proposition** is formed from existing propositions using logical operators such as $\neg$, $\vee$, $\wedge$, $\rightarrow$, and $\leftrightarrow$.

Negation

Definition: Let p be a proposition. The **negation** of p, denoted by $\neg p$, is the statement "It is not the case that p".

$\neg p$ is read as "not p." The truth value of $\neg p$ is the opposite of the truth value of p.

Examples:

$p =$ Sam uses an Android phone.

$\neg p =$ It is not the case that Sam uses an Android phone.

$\neg p =$ Sam does not use an Android phone. *(more simply expressed negation.)*

$q =$ Emma's PC has at least 256 GB of memory.

$\neg q =$ It is not the case that Emma's PC has at least 256 GB of memory.

$\neg q =$ Emma's PC has less than 256 GB of memory. *(more simply expressed negation.)*

Truth Table

Definition: A **truth table** of a compound proposition is a structured representation that presents all **possible combinations of truth values** of propositions used in that compound proposition and the **corresponding truth value** of it.

Truth Table of negation (¬):

p	$\neg p$
T	F
F	T

Conjunction

Definition: Let p and q be propositions. The **conjunction** of p and q, denoted by $p \wedge q$, is the statement "p and q".

The conjunction "p and q" is true when both p and q are true and is false otherwise.

Example:

p = Sam uses an Android phone.

q = Sam uses a Macbook.

$p \wedge q$ = Sam uses an Android phone **and** Sam uses a Macbook.

Disjunction

Definition: Let p and q be propositions. The **disjunction** of p and q, denoted by $p \vee q$, is the statement "p or q".

The disjunction "p or q" is false when both p and q are false and is true otherwise.

Example:

$$p = \text{Sam uses an Android phone.}$$

$$q = \text{Sam uses a Macbook.}$$

$$p \vee q = \text{Sam uses an Android phone } \textbf{or} \text{ Sam uses a Macbook.}$$

Note: Disjunction corresponds to "inclusive or" of English, not "exclusive or".

Students who have taken calculus **or** analysis can take this class.

Soup **or** salad comes with the main course.

Truth Tables of Conjunction and Disjunction

Truth Table of conjunction (∧)

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Truth Table of disjunction (∨)

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

Conditional Statements

Definition: Let p and q be propositions. The **conditional statement**, denoted by $p \to q$, is the statement "if p, then q".

p is called the **hypothesis** and q is called the **conclusion**.

Truth Table of conditional statement ($\to$)

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

Conditional Statements

Example 1:

$p =$ Alice scores 100% in Major.

$q =$ Alice gets an A.

$p \rightarrow q =$ **If** Alice scores 100% in Major, **then** Alice gets an A.

Example 2:

$p =$ Jack inserts a coin of 5 in vending machine.

$q =$ Vending machine gives a chocolate.

$p \rightarrow q =$ **If** Jack inserts a coin of 5 in vending machine, **then** vending machine gives a chocolate.

Conditional Statements

Some more ways to express $p \to q$.

- If p, then q.
- q if p.
- p is sufficient for q.
- p implies q.
- q is necessary for p.
- Etc.

Is the below proposition true? Yes.

If 5 is a prime number, then $2+2=4$.

Note: Conditional statements in logic do not have a cause-and-effect relationship.

Conditional Statements

Definition: For a proposition $p \to q$:

- $q \to p$ is called the **converse** of $p \to q$.
- $\neg q \to \neg p$ is called the **contrapositive** of $p \to q$.
- $\neg p \to \neg q$ is called the **inverse** of $p \to q$.

p	q	$p \to q$	$q \to p$	$\neg p$	$\neg q$	$\neg p \to \neg q$	$\neg q \to \neg p$
F	F	T	T	T	T	T	T
F	T	T	F	T	F	F	T
T	F	F	T	F	T	T	F
T	T	T	T	F	F	T	T

Same truth values.

Same truth values.